ILLINOIS LICENSURE TESTING SYSTEM

FIELD 202: MIDDLE GRADES (5–8) MATHEMATICS

TEST FRAMEWORK

June 2016

Effective beginning February 20, 2017

ILLINOIS LICENSURE TESTING SYSTEM

FIELD 202: MIDDLE GRADES (5–8) MATHEMATICS

TEST FRAMEWORK

June 2016

	Subarea	Range of Objectives
I.	Core Content	0001–0002
II.	Fractions, Ratios, and the Number System	0003–0004
III.	Relations, Functions, and Algebra	0005–0008
IV.	Measurement and Geometry	0009–0010
V.	Probability and Statistics	0011–0012
VI.	Disciplinary Literacy in Mathematics	0013–0016

Effective beginning February 20, 2017

ILLINOIS LICENSURE TESTING SYSTEM

FIELD 202: MIDDLE GRADES (5–8) MATHEMATICS

TEST FRAMEWORK

Core Content
Fractions, Ratios, and the Number System
Relations, Functions, and Algebra
Measurement and Geometry
Probability and Statistics
Disciplinary Literacy in Mathematics

SUBAREA I—CORE CONTENT

0001 Understand calculus.

For example:

- Demonstrate knowledge of factoring techniques, solving polynomial equations, and performing operations with rational and radical expressions.

- Demonstrate knowledge of different representations (e.g., tabular, algebraic, graphic) of polynomial, radical, and rational functions and circles; how to find horizontal and vertical asymptotes; and how to find points of intersection of curves.

- Apply the properties of limits to functions and their graphs.

- Apply the concept of continuity to functions and their graphs.

- Apply the rules of derivatives for finding tangent lines, slopes, rates of change, velocity and acceleration, marginal analysis, and increasing and decreasing functions.

- Apply concepts of differential calculus to model real-world situations.

- Demonstrate knowledge of the properties of exponential and logarithmic functions, graphs, and derivatives.

- Demonstrate knowledge of integration and basic antiderivatives and how to interpret integration as the area under a curve.

0002   Understand college statistics.

For example:

- Construct, identify, or interpret frequency distributions, histograms, box plots, and cumulative frequency tables and associated graphs.

- Identify, calculate, or interpret measures of central tendency (e.g., mean, median) and dispersion (e.g., interquartile range, standard deviation).

- Model and solve problems using the concept of expected value.

- Demonstrate knowledge of the probability distributions (e.g., normal, uniform, binomial) that random variables generate.

- Demonstrate knowledge of the sampling distribution of sample means and sample proportions.

- Demonstrate knowledge of confidence intervals and hypothesis testing for single population means and proportions.

- Identify, calculate, or interpret correlation coefficients and regression equations.

SUBAREA II—FRACTIONS, RATIOS, AND THE NUMBER SYSTEM

0003   Understand ratios and proportional relationships.

For example:

- Represent fractions and arithmetic operations on fractions using a variety of models (e.g., number lines, area models, diagrams).

- Apply properties of number operations to perform operational procedures with fractions, including the use of equivalent fractions.

- Solve problems using fractions, ratios, rates, and proportional relationships.

- Represent how quantities vary together in a proportional relationship, using tables, double number lines, and tape diagrams.

- Analyze the relationships between fractions, terminating decimals, and nonterminating decimals.

- Select or analyze instructional and assessment activities, methods, or strategies that use appropriate instructional materials or technology and allow students to develop understanding of mathematical practices, concepts, and skills.

0004   Understand the structure and properties of the real number system.

For example:

- Apply knowledge of the place value system to represent and perform operations on real numbers in a variety of ways (e.g., numerical expressions, diagrams, objects).

- Apply knowledge of number properties to solve problems using various representations of numbers (e.g., integers, exponents, scientific notation).

- Analyze and justify a variety of algorithms and computational strategies, including mental math approaches and techniques.

- Apply knowledge of prime numbers, factors, and divisibility to model and solve problems.

- Apply knowledge of greatest common factors and least common multiples to model number theory concepts and solve mathematical and real-world problems.

- Apply knowledge of the structure of the real number system, integer and rational exponents, irrational numbers, and radicals to model and solve problems.

- Select or analyze instructional and assessment activities, methods, or strategies that use appropriate instructional materials or technology and allow students to develop understanding of mathematical practices, concepts, and skills.

SUBAREA III—RELATIONS, FUNCTIONS, AND ALGEBRA

0005   Understand expressions and equations.

For example:

- Identify one- and two-variable equations and inequalities that represent a variety of situations or representations.

- Identify equivalent forms of mathematical expressions or equations that result from application of a variety of algebraic techniques (e.g., simplifying, factoring, solving for a different variable).

- Justify algebraic techniques using the properties of the real numbers.

- Evaluate the validity of proposed solutions to inequalities, equations, or systems of equations.

- Select or analyze instructional and assessment activities, methods, or strategies that use appropriate instructional materials or technology and allow students to develop understanding of mathematical practices, concepts, and skills.

0006   Understand relations and functions.

For example:

- Distinguish between relations and functions using a variety of representations.

- Analyze various representations (e.g., tabular, algebraic, graphic) of functions with respect to their characteristics (e.g., domain, intercepts, maxima and minima).

- Translate between representations of graphs, tables, and real-world situations or equations.

- Identify the effects of transformations such as $f(x + k)$, $f(x) + k$, and $k(f(x))$ on the graph of a function.

- Identify patterns of change in proportional, linear, inversely proportional, quadratic, and exponential functions and the types of real-world relationships that these functions can model.

- Identify expressions, equations, or functions that model specific situations.

- Select or analyze instructional and assessment activities, methods, or strategies that use appropriate instructional materials or technology and allow students to develop understanding of mathematical practices, concepts, and skills.

0007   Understand linear functions and relations.

For example:

- Analyze the relationship between the equation of a line and its graph in mathematical and real-world contexts.

- Determine the equation of a line using different types of information (e.g., two points, slope and one point).

- Identify the connection between proportional and linear relationships.

- Model and solve problems using different representations of linear equations and inequalities, and systems of linear equations and inequalities.

- Select or analyze instructional and assessment activities, methods, or strategies that use appropriate instructional materials or technology and allow students to develop understanding of mathematical practices, concepts, and skills.

0008   Understand nonlinear functions.

For example:

- Distinguish between linear and nonlinear relationships in a variety of situations (e.g., tables, real world).

- Translate between different representations (e.g., tabular, algebraic, graphic) of quadratic and exponential functions.

- Model and solve problems involving quadratic equations and inequalities using a variety of techniques (e.g., completing the square, factoring, graphing, quadratic formula).

- Model and solve problems involving exponential equations.

- Select or analyze instructional and assessment activities, methods, or strategies that use appropriate instructional materials or technology and allow students to develop understanding of mathematical practices, concepts, and skills.

SUBAREA IV—MEASUREMENT AND GEOMETRY

0009 Understand the principles of two- and three-dimensional geometry.

For example:

- Apply knowledge of quantities and units to convert measurements within and between various measurement systems.

- Apply formulas to find measures (e.g., angle, area, volume) of a variety of two- and three-dimensional figures in mathematical and real-world problems.

- Apply the Pythagorean theorem and its converse to solve mathematical and real-world problems.

- Use the properties of lines (e.g., parallel, perpendicular) and angles (e.g., supplementary, vertical) to characterize geometric relationships and solve problems.

- Apply the principles of congruence, similarity, and proportional and spatial reasoning (e.g., scale drawings) to solve mathematical and real-world problems.

- Analyze proofs of geometric theorems.

- Select or analyze instructional and assessment activities, methods, or strategies that use appropriate instructional materials or technology and allow students to develop understanding of mathematical practices, concepts, and skills.

0010 Understand the principles of coordinate and transformational geometries.

For example:

- Apply connections between geometric and algebraic properties (e.g., distance formula, midpoint formula, slope) to model and solve mathematical and real-world problems.

- Use two- and three-dimensional coordinate systems to represent and analyze the characteristics of various geometric figures.

- Analyze geometric transformations (e.g., translations, reflections, dilations, rotations) in a coordinate plane and their effects on congruence and similarity.

- Apply the properties of polygons (e.g., numbers and lengths of sides, measures of angles) to analyze and solve problems.

- Select or analyze instructional and assessment activities, methods, or strategies that use appropriate instructional materials or technology and allow students to develop understanding of mathematical practices, concepts, and skills.

Effective beginning February 20, 2017

SUBAREA V—PROBABILITY AND STATISTICS

0011   Understand the principles and techniques of probability.

For example:

- Apply methods of counting, including the difference between combinations and permutations, to represent and solve problems.

- Identify theoretical or experimental probabilities of simple, compound, independent, and conditional events using a variety of approaches (e.g., addition and multiplication rules).

- Apply the concept of sample space and its role in determining probabilities.

- Use and interpret a variety of representations of probabilities (e.g., Venn diagrams, tree diagrams).

- Select or analyze instructional and assessment activities, methods, or strategies that use appropriate instructional materials or technology and allow students to develop understanding of mathematical practices, concepts, and skills.

0012   Understand the principles and techniques of statistics.

For example:

- Apply random sampling techniques to collect representative data.

- Select appropriate data displays for a given situation (e.g., table, scatter plot).

- Interpret data in a variety of graphic formats (e.g., circle graphs, box plots, histograms).

- Analyze patterns in bivariate data and two-way frequency tables.

- Apply knowledge of measures of central tendency (e.g., mean, median), dispersion (e.g., interquartile range, standard deviation), and shapes of data distributions.

- Select or analyze instructional and assessment activities, methods, or strategies that use appropriate instructional materials or technology and allow students to develop understanding of mathematical practices, concepts, and skills.

SUBAREA VI—DISCIPLINARY LITERACY IN MATHEMATICS

0013   Apply knowledge of foundations of research-based disciplinary literacy instruction and assessment.

For example:

- Apply knowledge of appropriate research to identify and implement instructional practices and strategies that are effective in supporting the disciplinary literacy development of all students.

- Apply knowledge of the developmental sequence of language and literacy skills for adolescent learners, and analyze the role of systematic and explicit instruction in the development of disciplinary literacy skills.

- Analyze the nature and communicative role of various features of language (e.g., semantics, syntax, discourse) in disciplinary literacy development.

- Apply knowledge of the use of a wide range of high-quality informational texts to support the development of disciplinary literacy (e.g., selecting appropriate, high interest, and culturally responsive texts; using a variety of media).

- Apply knowledge of the use of assessment to monitor students' development of disciplinary literacy (e.g., using continuous monitoring of student progress to plan and evaluate disciplinary literacy instruction, providing feedback to students on their work to help them understand their own progress and how to improve performance, engaging students in self-assessment of their development in disciplinary literacy).

- Analyze strategies and routines that contribute to the development of a supportive language and literacy environment in the mathematics classroom (e.g., promoting self-direction that leads to independence and ownership, building collaborative classroom communities).

0014   Apply knowledge of academic-language and vocabulary development to support students' disciplinary literacy development in the mathematics classroom.

For example:

- Analyze the role of language structures, mathematical symbols, and vocabulary in supporting students' understanding of mathematical concepts, content, skills, and processes.

- Apply knowledge of criteria for selecting vocabulary for explicit word study in the mathematics classroom (e.g., words central to a unit of study and/or the meaning of a text and likely to be unfamiliar to students).

- Apply knowledge of a wide variety of strategies for developing and expanding students' depth of understanding and retention of new mathematics vocabulary.

- Apply knowledge of strategies for promoting students' understanding of and ability to use various forms (e.g., sentence structures, text structures) and functions (e.g., interpreting, classifying, justifying) of language to develop and express content understanding in the mathematics classroom.

- Apply knowledge of strategies for promoting students' ability to analyze, interpret, and use conventions of Standard American English grammar and usage to support their listening comprehension, speaking, reading comprehension, and writing in the mathematics classroom.

- Apply knowledge of differentiated instruction and appropriate assessment strategies in mathematical language and vocabulary development that are responsive to the diverse strengths and needs of all students.

0015 Apply knowledge of reading comprehension to support students' disciplinary literacy development in the mathematics classroom.

For example**:**

- Apply knowledge of the organizational structures, rhetorical features, text features, and graphics commonly used in texts in the mathematics classroom, including the characteristics of various forms of informational texts and the role, perspective, and purpose of various texts.

- Analyze text features that may impede students' reading comprehension (e.g., use of vocabulary, complexity of sentences, use of data).

- Apply fundamental principles for instruction to enable students to understand and learn from challenging text, using various reading strategies to improve comprehension (e.g., visualizing, questioning, synthesizing, evaluating) and providing explicit instruction in note-taking and text annotation.

- Apply knowledge of strategies for promoting students' ability to trace and evaluate the argument and specific claims in texts and to distinguish claims that are supported by reasons and evidence from claims that are not supported.

- Apply knowledge of strategies for promoting students' ability to interpret graphic features of texts (e.g., tables, charts, illustrations, captions, headings) and determine their relationship to the text (e.g., how well information in a graphic feature aligns with informational text).

- Apply knowledge of strategies for promoting students' ability to analyze texts (e.g., chronological, sequential, cause/effect, compare/contrast) and recognize features of text common to mathematics.

- Apply knowledge of differentiated instruction and appropriate assessment strategies in reading comprehension that are responsive to the diverse strengths and needs of all students.

0016 Apply knowledge of the development of writing, listening, and speaking skills to support students' disciplinary literacy development in the mathematics classroom.

For example:

- Apply fundamental principles for instruction in writing, listening, and speaking in the mathematics classroom, including providing instructional support and opportunities for students to write, listen, and speak routinely for authentic purposes; providing feedback on written work and oral presentations to guide students' revisions; engaging students in writing and oral language activities to develop their understanding of mathematics concepts and skills (e.g., participating in collaborative writing and discussions about mathematics, asking mathematics and text-based questions, reporting on a topic, recounting experiences related to mathematics learning); and engaging students in using technology to produce and publish mathematics writing and to interact and collaborate with others about mathematics.

- Apply knowledge of strategies for promoting students' ability to present ideas and information effectively in the mathematics classroom and to produce coherent and clear writing and oral presentations that reflect organization, development, substance (e.g., relevant facts and details), transitional devices, style, and use of technology (e.g., presentation software, media and visual displays) appropriate to the task, purpose, and audience.

- Apply knowledge of strategies for promoting students' ability to develop arguments and claims that are supported by valid reasoning and sufficient, relevant evidence; and knowledge of narrative texts that use description and organize a sequence of events.

- Apply knowledge of strategies for promoting students' ability to conduct mathematics research projects, including selecting and developing topics; gathering information from a variety of sources; assessing the credibility and accuracy of sources; synthesizing information; and avoiding plagiarism through the use of appropriate paraphrasing and summarizing and by quoting or citing sources following a standard format for citations.

- Apply knowledge of differentiated instruction and appropriate assessment strategies in writing, listening, and speaking that are responsive to the diverse strengths and needs of all students.